

# A fully nonlinear Dirichlet principle

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## Abstract

The  $k$ -Hessian operator  $\sigma_k$  is the  $k$ -th elementary symmetric function of the eigenvalues of the Hessian. It is known that the  $k$ -Hessian equation  $\sigma_k(D^2u) = f$  with Dirichlet boundary condition  $u = 0$  is variational; indeed, this problem can be studied by means of the  $k$ -Hessian energy  $\int -u\sigma_k(D^2u)$ . We construct a natural boundary functional which, when added to the  $k$ -Hessian energy, yields as its critical points solutions of  $k$ -Hessian equations with general non-vanishing boundary data. As a consequence, we prove a sharp Sobolev trace inequality for  $k$ -admissible functions  $u$  which estimates the  $k$ -Hessian energy in terms of the boundary values of  $u$ . This is joint work with Jeffrey Case.