Title: On the Tamagawa number conjecture for CM elliptic curves.

Abstract

The Tamagawa number conjecture for motives or the Bloch-Kato conjecture on special values of L-functions [1] [3] [2] [5], predicts for every motive M the exact value of its L-function at 0. If we restrict to a Chow motive of an abelian variety A of the form $h^i(A)$, the Tamagawa number conjecture in this situation predicts, all the integer values of the L-function associated to $h^i(A)$, twisting the above motive by Tate twists. We consider the case of elliptic curves E with CM by the ring of integers $\mathcal{O}_k$ of an imaginary quadratic field K. The conjecture for the motive $h^1(E)(1)$, that is for the integer value in the critical band, is equivalent to the Birch-Swinnerton-Dyer conjecture. We concentrate in the non-critical band. Kato studied the case $h^1(E)(2)$ when E is already defined over $\mathbb{Q}$ in [1]. Kings proved in [4] the case $h^1(E)(k)$ with $k \geq 2$ when E is defined over K using the spezialization of the elliptic polylogarithm, joint with the main lwasawa conjecture for imaginary quadratic fields proved by Rubin in [6]. We proof the case when E is defined over $\mathbb{Q}$ by a descent argument.

We plan to explain the $\mathbb{Q}$-coefficient conjecture to apply it at $h^1(E)(k)$, for $k \geq 2$ and E defined over $\mathbb{Q}$, following Kato. In order to use the CM structure of E, is natural to study the conjecture for $h^1(E \times_{\mathbb{Q}} K)(k)$, relating it to the K-structures we get in all the realizations. Then we obtain the result for $h^1(E)(k)$ via descents arguments.

References

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