

Collapsing behaviour of the logarithmic diffusion equation

Kin Ming Hui

Institute of Mathematics, Academia Sinica,
Nankang, Taipei, 11529, Taiwan, R. O. C.

Sept 22, 2012

Abstract

Let $0 \leq u_0(x) \in L^1(\mathbb{R}^2) \cap L^\infty(\mathbb{R}^2)$ be such that $u_0(x) = u_0(|x|)$ for all $|x| \geq r_1$ and is monotone decreasing for all $|x| \geq r_1$ for some constant $r_1 > 0$ and $\text{ess inf}_{\bar{B}_{r_1}(0)} u_0 \geq \text{ess sup}_{\mathbb{R}^2 \setminus B_{r_2}(0)} u_0$ for some constant $r_2 > r_1$. Then under some mild decay conditions at infinity on the initial value u_0 we will extend the result of P. Daskalopoulos, M.A. del Pino and N. Sesum [?], [?], and prove the collapsing behaviour of the maximal solution of the logarithmic diffusion equation $u_t = \Delta \log u$ in $\mathbb{R}^2 \times (0, T)$, $u(x, 0) = u_0(x)$ in \mathbb{R}^2 , near its extinction time $T = \int_{\mathbb{R}^2} u_0 dx / 4\pi$ without using the Hamilton-Yau Harnack inequality.

Key words: collapsing behaviour, maximal solution, logarithmic diffusion equation

References

- [1] P. Daskalopoulos and M.A. del Pino, *Type II collapsing of maximal solutions to the Ricci flow in \mathbb{R}^2* , Ann. Inst. H. Poincaré Anal. Non Linéaire 24 (2007), 851–874.
- [2] P. Daskalopoulos and N. Sesum, *Type II extinction profile of maximal solutions to the Ricci flow equation*, J. Geom. Anal. 20 (2010), no. 3, 565–591.
- [3] K.M. Hui, *Collapsing behaviour of a singular diffusion equation*, Discrete and Continuous Dynamical Systems Series A 32 (2012), no. 6, 2166–2185.